James Ruse Agricultural High School 2010 Year 12 Mathematics Trial Exam

Question 1. **Marks**

(a) Evaluate to 2 significant figures : $\frac{3.72\times1.96+\sqrt{4.3+2.7^2}}{3.6\times1.8+3.1^3}$ 1

(b) Rationalise the denominator and write in the form $a + b\sqrt{2}$: $\frac{3\sqrt{2}+4}{2\sqrt{2}-3}$ 2
where a, b are real.

(c) Find the acute angle (to the nearest minute) that the line $4x - 11y + 9 = 0$ makes with the *x* axis. 2

(d) Graph $y = 2sin3x$ in the domain $-\pi \leq x \leq \pi$. 2

(e) Find $\lim_{h\to0}\left(\frac{4^h-1}{2^h-1}\right)$ 2

(f) Solve : $|x - 3| = 4x + 2$ 3

Question 2.

Three points *A*,*B* and *C* lie on the *x*-*y* plane.
The lines *l* and *k* represent the lines *AB* and *AC* respectively.
The equations of lines *l* and *k* are respectively:

$3x - 4y - 100 = 0$ and $16x - 63y + 175 = 0$ respectively.

(a) Show that *B(8,-19)* lies on the line *l.* 1

(b) Find the co-ordinates *A* of the intersection of lines *l* and *k.* 3

(c) Find in general form the equation of the line *m* perpendicular to line *l* passing through *B.* 2

(d) Show that line *m* intersects line *k* at the point *C(-7, 1).* 2

(e) Find the exact perpendicular distance of *B* from *AC.* 2

(f) Find the area of triangle *ABC.* 2

Question 3. **Marks**

(a) Differentiate : (i) $\frac{3}{\sqrt{1-2x}}$ 2

(ii) $\frac{sinx}{x}$ 2

(iii) e^{tanx} 2

(b) Find (i) $\int \sqrt{e^{2x}}\, dx$ 1

(ii) $\int (\cot x - cosec^2\, x\,)\, dx$ 2

(c) Find in simplest terms : $\frac{d}{dx}\{x^2(2\, lnx - 1)\}$, hence evaluate $\int_1^e x\, lnx\, dx$. 3

Question 4.

(a) Given the equation $x^2 = 16\, (\, y + 4\,)$

(i) State the co-ordinates of the vertex. 1

(ii) Find the focal length 1

(iii) State the co-ordinates of the focus 1

(iv) Find in general form the equation of the tangent at (-12, 5) 2

(v) Find the co-ordinates of the point where the tangent meets the directrix. 1

(b) A jar has 15 red discs and 9 black discs, while another jar has 20 red discs, 15 black discs and 10 white discs. 2
A disc is drawn from each jar.
Find the probability of drawing discs of the same colour ?

(c) A car tyre of diameter 60cm is in contact with the road at the point *P*. 4
After the car has travelled 1000km how high (to the nearest millimetre) is the point *P* from the ground.

Question 5.

(a) Given $N = x^{n-1} + x^{n-2}y \;\; + x^{n-3}y^2 + \ldots\ldots\ldots\ldots\ldots xy^{n-2} + y^{n-1}$

(i) Simplify *N* in terms of *x* and *y*. 2

(ii) Hence prove $11^{21} - 5^{21}$ is divisible by 3. 2

(b) Use Simpson's Rule with 3 function values to evaluate to 2 decimal places : 3

$$\int_0^2 \frac{4\, dx}{2sinx + 1}$$

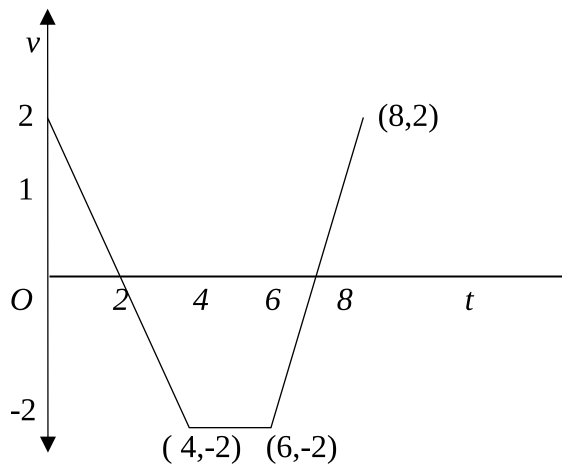
(c) Solve to 2 decimal places : $3^{2x+1} - 3^x = 10$ 3

(d) If the quadratic equation : $(k^2 + l^2)x^2 + 2l\,(k + m)x + l^2 + m^2 = 0$ has equal roots then show $l^2 = km$. 2

Question 6. **Marks**

(a) The region bounded by the curve $y = x\,(6 - x)$ and $y = 8$ is rotated around the x axis. 4
Find the exact value of the Volume of revolution.

(b)



A particle of mass 2 kg moves in a straight line with velocity *v m/s* and displacement *x m* at time *t* seconds.

(i) Graph acceleration $\ddot{x}\ m/s^2$ versus time *t* seconds. 2

(ii) Find the total distance travelled during the motion. 1

(c) Find in general form the equation of the inflexional tangent on the curve **:** 5

$$y = 15 + 12x + 6x^2 - 2x^3$$

Question 7.

(a) (i) On the same axes graph **:** 4

(α) the line $y = 1 - 2x$ showing *x* and *y* intercepts.

(β) the curve $y = 5 - 2x - x^2$,
showing the co-ordinates of the vertex and *y* intercept only.

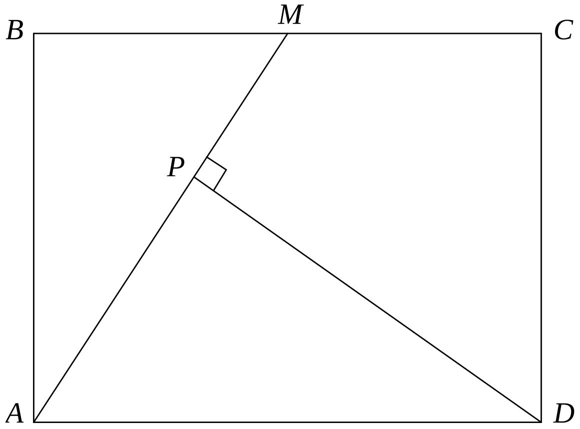
(ii) Find the *x* values of the points *A* and *B* of the intersection of the line $y = 1 - 2x$ and the curve $y = 5 - 2x - x^2$. 2

(iii) Evaluate the enclosed area between the line $y = 1 - 2x$ and the curve $y = 5 - 2x - x^2$. 3

(b) The rate of decay $\frac{dM}{dt}$ of a radioactive substance is proportional to the mass *M* present. 3
If it takes 51 minutes to decay to $\frac{1}{10}$ of it's original mass find the half-life of the substance (nearest minute).

Question 8. **Marks**

(a)



ABCD is a rectangle in which *AB*=40cm and *AD*=60 cm.
M is the midpoint of *BC* and *DP* is perpendicular to *AM*.

Draw a neat sketch of the above diagram.

(i) Prove that triangles *ABM* and *APD* are similar. 2

(ii) Calculate the length of *PD*. 2

(iii) Show that the length of *AP* is 36 cm. Give reasons. 2

(iv) Find the area of the quadrilateral *PMCD*. 3

(b) A plane flies from town *O* to town *A*, 275 km on a bearing of 032^0T, then to town *B* 572 km on a bearing of $S\ 26^0E$. 3

(i) Draw a diagram to show the above information.

(ii) Find the final distance (nearest km), and bearing (nearest degree) from *O*.

Question 9.

(a) A particle of mass *m* kg moves in a straight line with velocity *v m/s* and displacement *x* metres at time *t* seconds.

The velocity of the particle is given by **:** $v = 3\sqrt{1+9t}$.

Find (i) the acceleration $\ddot{x}$ in terms of time *t*. 1

(ii) the displacement of the particle as a function of time *t* if the particle is initially 1 metre to the **left** of the origin. 2

(b) A man buys a house and land for \$500 000. **Marks**
He pays 20% deposit, and takes a loan for the remainder.

(i) Find the value of the deposit. 1

(ii) If the loan is for 20 years, and the interest rate is 8% p.a. monthly reducible show that the amount owing after the first monthly repayment R is **:** 1

$$\$\ (400\ 000(\tfrac{151}{150}) - R\ )$$

(iii) Find the amount owing after n months. 2

(iv) Find the monthly repayment. 2

(v) Find the amount owing after the 144th payment. 1

(vi) The value of the land was originally valued at \$270 000 . If the value of the land was compounded yearly at 6% p.a. find the value of the land after the 144th payment. 1

(vii) After the 144th payment an earthquake destroys the house. 1
The insurance policy does not cover earthquakes.
Could the man sell the land to pay the remainder of the loan?
Give reasons.

Question 10.

A series S is given by **:**

$$S = x + \frac{2x^2}{x+1} + \frac{4x^3}{(x+1)^2} + \frac{8x^4}{(x+1)^3} + \ldots\ldots\ldots\ldots\ldots\ldots..$$

(a) Sketch the curve $y = \dfrac{2x}{x+1}$, showing all asymptotes and intercepts with the axes. 2

(b) Find the values of x for the sum to infinity to exist. 2

(c) Show that the sum to infinity is given by **:** 2

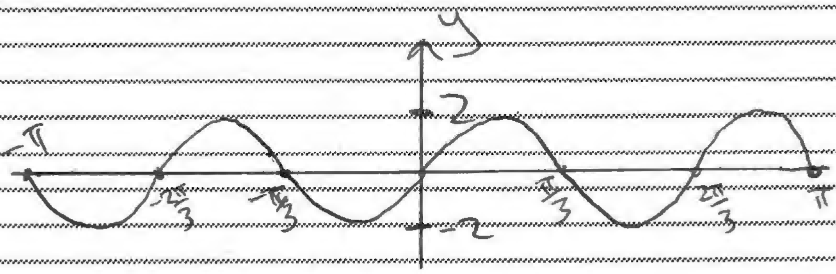
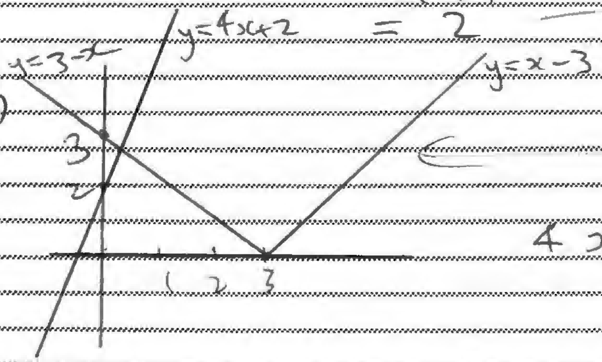
$$S_\infty = \frac{x^2+x}{1-x}$$

(d) Show that $\dfrac{dS_\infty}{dx} = \dfrac{-x^2+2x+1}{(1-x)^2}$ 2

(e) Find the minimum value of the sum to infinity. 4
Justify your answer.

End of Exam

2u **MATHEMATICS: Question** 1

Suggested Solutions	**Marks**	**Marker's Comments**
(a) $\dfrac{3.72\times 1.96+\sqrt{4.3+2.7^2}}{3.6\times 1.8+3.1^3}$ $= 0.29488$ $= 0.29$ (2 sig figures)	$\frac{1}{2}$ $\frac{1}{2}$	If they write out the calculator display and it was wrong but they rounded up correctly they get ½ mk.
(b) $\dfrac{(3\sqrt{2}+4)}{(2\sqrt{2}-3)}\dfrac{(2\sqrt{2}+3)}{(2\sqrt{2}+3)} = \dfrac{12+9\sqrt{2}+8\sqrt{2}+12}{8-9}$ ½ $= -24-17\sqrt{2}$	½ ½ ½	
(c) $m = 4/11$ $\tan\theta = \frac{4}{11}$ $\theta = 19^\circ 59'$ (nearest minute)	1 ½ ½	
(d) 		½ _ intercepts ½ - shape ½ - amplitude ½ - period
(e) $\lim\limits_{h\to 0}\dfrac{(4^h-1)}{2^h-1} = \lim\limits_{h\to 0}\dfrac{(2^h-1)(2^h+1)}{2^h-1}$ $= \lim\limits_{h\to 0}(2^h+1)$ $= 1+1$ $= 2$	1 ½ ½	an answer of "2" with no working = ½ mk.
(f) $4x+2 = 3-x$ $5x = 1$ $x = 1/5$	2mks 1mk	* scored 2 mks if you solved algebraically but forgot to check the answers.

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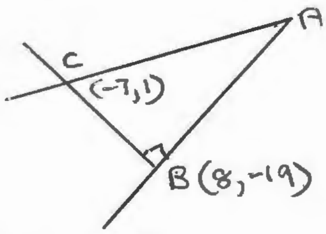
2010 TRIAL 2U **MATHEMATICS: Question** 2

Suggested Solutions	Marks	Marker's Comments
2 (a) Show $B(8,-19)$ lies on $3x-4y-100=0$ $LHS = 3\times 8 - 4\times -19 - 100$ $= 24+76-100$ $= 0$ $\therefore LHS = RHS$, hence $B(8,-19)$ lies on ℓ.	1	$0=0$ without saying $LHS = RHS$ lost $\frac{1}{2}$ mark
(b) $\ell: 3x-4y-100=0 \ldots (1)$ $k: 16x-63y+175=0 \ldots (2)$ $(1)\times 16$ and $(2)\times 3$ $48x-64y=1600 \ldots (3)$ $48x-189y=-525 \ldots (4)$ $(3)-(4)$ $125y=2125$ $y=17$ Subst. into (1) $3x-68=100$ $3x=168$ $x=56$ $\therefore A(56,17)$	1 1 1	1 mark deducted for each error
(c) $m_\ell = \frac{3}{4}$ Gradient of a line perpendicular to ℓ is $-\frac{4}{3}$ as $m_1\times m_2=-1$ for perp. lines Eqn is: $y+19=-\frac{4}{3}(x-8)$ $3y+57=-4x+32$ $\therefore$ Eqn. of the line in general form is: $4x+3y+25=0$	1 1	Not in general form, max of 1 mark only
(d) For C $4x+3y=-25 \ldots (1)$ $16x-63y=-175 \ldots (2)$ $(1)\times 4$ $16x+12y=-100 \ldots (3)$ $(3)-(2)$ $75y=75$ $y=1 \ldots (4)$ Sub (4) in (1) $4x+3=-25$ $4x=-28$ $x=-7$ $\therefore C(-7,1)$	1 1	

2010 TRIAL 2U **MATHEMATICS: Question** 2

Suggested Solutions	Marks	Marker's Comments
2(e) $D = \left\| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right\|$ $= \dfrac{16\times 8 - 63\times -19 + 175}{\sqrt{16^2+63^2}}$ $= \dfrac{1500}{65} = \dfrac{300}{13}$ $\therefore D = 23\frac{1}{13}$ units	 1 1	No penalty for units missing
(f)  $AC = \sqrt{63^2+16^2}$ $= 65$ units $\therefore$ Area $\Delta ABC = \frac{1}{2} \times 65 \times 23\frac{1}{13}$ $A = 750$ units^2	 1 1	

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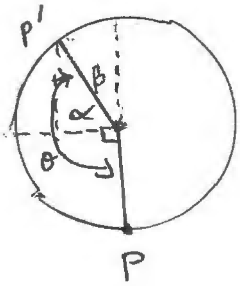
Mathematics - 2 unit

Question 3	
a) Differentiate i) $\frac{3}{\sqrt{1-2x}} = \frac{d}{dx} 3(1-2x)^{-\frac{1}{2}} = 3 \times -\frac{1}{2} \cdot -2(1-2x)^{-\frac{3}{2}}$ ✓ $= 3(1-2x)^{-\frac{3}{2}}$ ✓ OR $\frac{3}{(1-2x)^{3/2}}$ or $\frac{3}{(1-2x)(\sqrt{1-2x})}$	$\frac{-3}{2\sqrt{(1-2x)^3}}$ earned 1 mark $\frac{-3}{\sqrt{1-2x}}$ earned 1 mark 2 marks
ii) $\frac{d}{dx} \frac{\sin x}{x} = \frac{x\cos x - \sin x}{x^2}$ ✓ ✓	$1\frac{1}{2}$ mark if $\frac{\sin x - x\cos x}{x^2}$ $\boxed{2 \text{ marks}}$
iii) $\frac{d}{dx} e^{\tan x} = \sec^2 x\, e^{\tan x}$ ✓ ✓	$\boxed{2 \text{ marks}}$
b (i) $\int \sqrt{e^{2x}}\, dx = \int e^{x}\, dx$ $= e^{x} + C$ ✓ $\boxed{1 \text{ mark}}$	$-\frac{1}{2}$ if no C Note cannot apply chain rule for $\int$.
(ii) $\int (\cot x - \operatorname{cosec}^2 x)\, dx = \int \left(\frac{\cos x}{\sin x} - \operatorname{cosec}^2 x\right) dx$ $\int \frac{\cos x}{\sin x}\, dx = \ln \sin x + C$ ✓ we know the derivative of $\cot x = -\operatorname{cosec}^2 x$ proof $\cot x = \frac{\cos x}{\sin x}$ let $u = \cos x \to u' = -\sin x$ $v = \sin x \to v' = \cos x$ Using Quotient Rule $\frac{\sin x(-\sin x) - \cos^2 x}{\sin^2 x}$ $= \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$ $= \frac{-1}{\sin^2 x}$ $= -\operatorname{cosec}^2 x$ $\therefore \int -\operatorname{cosec}^2 x \,. dx = \cot x + c$ ✓ $\therefore \int \cot x - \operatorname{cosec}^2 x = \ln \sin x + \cot x + C$	$-\frac{1}{2}$ wrong sign $\boxed{2 \text{ marks}}$
c) $\frac{d}{dx} x^2 (2\ln x - 1) = x^2 \cdot \frac{2}{x} + (2\ln x - 1)2x$ $= 4x\ln x$ ✓ $\int_1^e x\ln x\, dx = \left\{\frac{1}{4} x^2 [2\ln x - 1]\right\}_1^e$ ✓ $= \frac{1}{4}\left[e^2(2-1) - 1.(-1)\right]$ $= \frac{e^2+1}{4}$ ✓	Use product rule $\boxed{3 \text{ marks.}}$

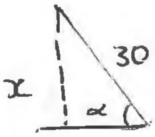
Question 4

a) (i) Vertex $(0,-4)$ ✓	1 mark
ii) $4a = 16$ $a = 4$ ✓	1 mark
iii) focus $(0,0)$ ✓	1 mark
iv) $y = \frac{x^2}{16} - 4$ $\frac{dy}{dx} = \frac{x}{8}$ at $x = -12$ $m = \frac{-12}{8}$ $m = \frac{-3}{2}$ ✓ Equation of tangent $y - y_1 = m(x - x_1)$ $y - 5 = -\frac{3}{2}[x + 12]$ $2y - 10 = 3x - 36$ $3x + 2y + 26 = 0$ ✓	few did not simplify $\frac{-12}{8} = \frac{-3}{2}$ 2 marks
v) Directrix $y = -8$ $3x + 2(-8) + 26 = 0$ $3x = -10$ $x = -3\frac{1}{3}$ $\therefore$ point is $(-3\frac{1}{3}, -8)$ ✓	1 mark
b) $p(R,R) + p(B\,B) = \frac{15}{24} \times \frac{20}{45} + \frac{9}{24} \times \frac{15}{45}$ $= \frac{5}{8} \times \frac{4}{9} + \frac{3}{8} \times \frac{3}{9}$ ✓ $= \frac{29}{72}$ ✓	2 marks
c) Circumference of Tyre $= 60\pi$ Distance travelled $= 1000$ km $= 1000000$ m $= 100000000$ cm $= 1\,000\,000\,000$ mm Number of revolutions $= \frac{\text{Distance travelled}}{\text{circumference tyre}}$ $= \frac{100000000 \text{ cm}}{60\pi \text{ cm}}$ $= 530516.477$ rev. There is 530,516 complete revolutions with 0.477 of a revolution left over Hence 0.477 of circumference (60π cm) is left over ie $0.477 \times 60\pi$ $= 89.91238175$ cm. Calculate θ $l = r\theta$ $89.91238175 = 30\theta$ $\theta = 2.997079392^c$ or 0.954π or 171.72°	• few $r = 60$ cm instead of 30 cm. • Did not change both distance and radium into same unit 4 marks



$\alpha = \theta - \frac{\pi}{2}$

$= 1.426283065$



$\sin\alpha = \frac{x}{30}$

$x = 30 \sin\alpha$

$\simeq 29.8$

$\therefore$ Total height $= 30 + x$

$= 30 + 29.68658014$

$= 59.68658014$

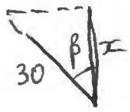
$= 597$ mm

$\boxed{\text{OR}}$

$\beta = \pi - \theta$

$= \pi - 171.72^{\circ}$

$= 8.28^{\circ}$



$\cos 8.28 = \frac{x}{30}$

$x = 30 \cos 8.28^{\circ}$

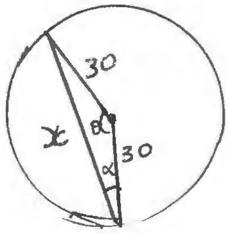
$= 29.68728356$

$\therefore$ height $\quad 29.68728356$

$+ 30$

$= 59.68728356$

height $= 597$ mm



$\alpha = \frac{180 - 171.72}{2}$

$= 4.14^{\circ}$

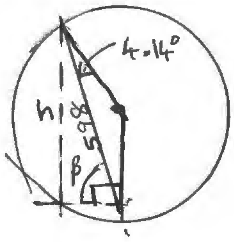
Sine Rule

$\frac{x}{\sin 171.72^{\circ}} = \frac{30}{\sin 4.14^{\circ}}$

$x = \frac{30 \sin 171.72^{\circ}}{\sin 4.14^{\circ}}$

$= 59.843$ cm

$= 598$ mm



$\beta = 90 - 4.14^{\circ}$

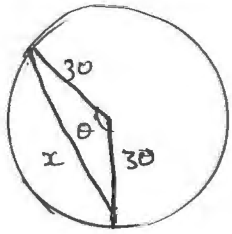
$= 85.86$

$\sin\beta = \frac{h}{598}$

$h = 598 \sin 85.86$

$h = 596.4395939$

$h = 596$ mm.



Cosine rule

$x^2 = 30^2 + 30^2 - 2(30)(30)\cos 0.954\pi$

$= 3581.237013$

$x = 59.843$

$= 598$ mm

Similarily

as in previous example

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Comments

Q5

a) $N = \dfrac{x^{n-1}\left[\left(\frac{y}{x}\right)^{n}-1\right]}{\frac{y}{x}-1}$ 1m

$N = \dfrac{x^n - y^n}{x-y}$ 1m

many students did not simplify completely to lowest term.

$x^n - y^n = (x-y)N$

$11^{21}-5^{21} = (11-5)\left[11^{20}+11^{19}\cdot 5+\dots+5^{20}\right]$ 1m

$= 6\times(11^{20}+11^{19}\cdot 5+\dots+5^{20})$

$= 3\times 2\times\underbrace{(11^{20}+11^{19}\cdot 5+\dots+5^{20})}_{\text{integer}}$ 1m

$\therefore 11^{21}-5^{21}$ is divisible by 3 #

many students did not justify $11^{20}+11^{19}\cdot 5+\dots+5^{20}$ is an integer and simply say N is an integer get 1m only.

b)

x	0	1	2
$\frac{4}{2\sin x+1}$	4	1.4909	1.4191

1m

$\int_0^2 \frac{4\,dx}{2\sin x+1} = \frac{2-0}{6}\left[4+4\left[\frac{4}{2\sin 1+1}\right]+\frac{4}{2\sin 2+1}\right]$ 1m

$= 3.79425$

$= \underline{\underline{3.79\ (2dp)}}$ 1m

c) $3\cdot 3^{2x}-3^{x}-10=0$

Put $u=3^x$

$3u^2-u-10=0$ 1m

$(3u+5)(u-2)=0$

$u=2$ or $u=-5/3$

$\therefore 3^x=2$ or $3^x=-5/3$ 1m

but $3^x>0$ $\therefore 3^x=2$ only

$x=\frac{\ln 2}{\ln 3}=0.6309\ldots$

$x=\underline{\underline{0.63\ (2dp)}}$ 1m

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5 d)

$\Delta = 0$ for equal roots

$[2l(k+m)]^2 - 4[k^2+l^2][l^2+m^2] = 0$ 1m

$4l^2[\cancel{k^2} + 2km + \cancel{m^2}] - 4[\cancel{k^2 l^2} + k^2m^2 + l^4 + \cancel{l^2 m^2}] = 0$

$\cancel{4}l^2 \cdot 2km - \cancel{4}k^2m^2 - \cancel{4}l^4 = 0$

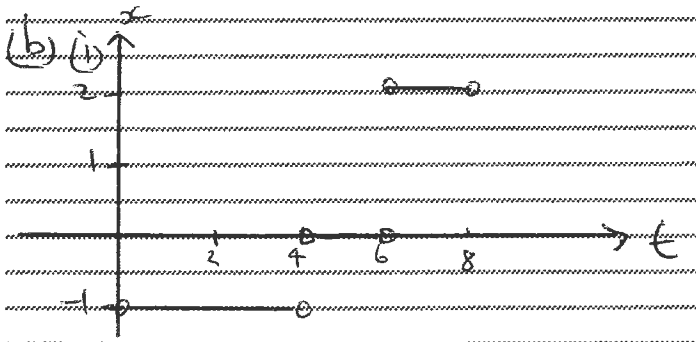
$l^4 + k^2m^2 - 2kml^2 = 0$

$(l^2 - km)^2 = 0$ 1m

$l^2 = km$ #

many students made mistakes or cannot complete square max 1m

2U MATHEMATICS: Question 6

Suggested Solutions	Marks	Marker's Comments
(a) intersection points: $x(6-x)=8$ $x^2-6x+8=0$ $(x-4)(x-2)=0$ $x=4$ or $x=2$	1	If they had the wrong limits, 1 mark off
Volume $=\pi\int_2^4 x^2(6-x)^2\,dx-\pi\int_2^4 8^2\,dx$	½	* 1½ mk off if they forgot to square the fns. * ½ mk off if the fns are around the wrong way
$=\pi\int_2^4 x^2(36-12x+x^2)\,dx-\pi\int_2^4 64\,dx$		* ½ mk off if squared as one big function
$=\pi\int_2^4 (36x^2-12x^3+x^4-64)\,dx$		* ½ mk off for every calculator error
$=\pi\left[12x^3-3x^4+\frac{x^5}{5}-64x\right]_2^4$	1	* 1 mk off if they forgot $\pi\int_2^4 8^2\,dx$.
$=\pi\left[\frac{4^5}{5}+12(4)^3-3(4)^4-64(4)\right]-\pi\left[12\times8-3(16)+\frac{32}{5}-128\right]$	1	
$=\pi\left(204\frac{4}{5}-768+768-256\right)-\pi\left(96-48+6\frac{2}{5}-128\right)$		
$=\pi\left(198\frac{2}{5}-176\right)$	½	
$=22\frac{2}{5}\pi$ or $\frac{112\pi}{5}$ units^3 or 22.4π units^3		
(b) (i) 	2 mks	* ½ mk off for no open circles * ½ mk off if lines joined up * 0 for curves!
(ii) Total distance $=\frac{2\times2}{2}+\frac{2\times2}{2}+2\times2+\frac{1\times2}{2}+\frac{1\times2}{2}$ $=2+2+4+1+1$ $=10$ metres	1	* ½ mk if they got 8 metres.
(c) $y=15+12x+6x^2-2x^3$ $\frac{dy}{dx}=12+12x-6x^2$	½	

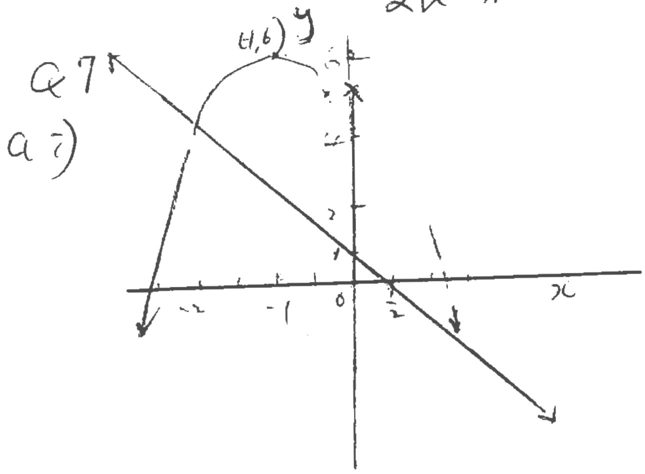
<table>
<tr><th>Suggested Solutions</th><th>Marks</th><th>Marker's Comments</th></tr>
<tr><td>$\frac{d^2y}{dx^2} = 12-12x$</td><td>$\frac{1}{2}$</td><td></td></tr>
<tr><td>possible points of inflexion when $\frac{d^2y}{dx^2}=0$</td><td>$\frac{1}{2}$</td><td></td></tr>
<tr><td>$12-12x=0$
$x=1$</td><td>$\frac{1}{2}$</td><td></td></tr>
<tr><td>when $x=1$, $y=15+12+6-2$
$y=31$ $(1,31)$</td><td>$\frac{1}{2}$</td><td></td></tr>
<tr><td>when $x=1$, $y'=12+12-6$
$=18$
$\therefore m=18$</td><td>$\frac{1}{2}$</td><td></td></tr>
<tr><td>test for change in concavity
<table><tr><td>x</td><td>0.9</td><td>1</td><td>1.1</td></tr><tr><td>$\frac{d^2y}{dx^2}$</td><td>1.2</td><td>0</td><td>-1.2</td></tr><tr><td>concavity</td><td>up</td><td>—</td><td>down</td></tr></table>$\therefore$ change in concavity at $x=1$
$\therefore$ $(1,31)$ is a point of inflexion</td><td>1</td><td>* ½ mk off if they didn't use numbers or if they didn't state theres a change in concavity</td></tr>
<tr><td>eqn. of inflexional tangent is:
$y-y_1=m(x-x_1)$
$y-31=18(x-1)$</td><td>$\frac{1}{2}$</td><td></td></tr>
<tr><td>$18x-y+13=0$</td><td>$\frac{1}{2}$</td><td></td></tr>
</table>

comments

2u Maths Trial 2010

Q7

a i)



Intercepts $\frac{1}{2}$m each

$(0,1)$, $(\frac{1}{2}, 0)$, $(0,5)$

vertex $(-1, 6)$ 1m

parabola 1m

straight line $\frac{1}{2}$m

generally well done.

ii) intersect at $x = \pm 2$ 2m

iii) Area $= \int_{-2}^{2} (5 - 2x - x^2) - (1 - 2x)\,dx$ 1m

$= \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$ 1m

$= 10\frac{2}{3}$ unit^2 1m

generally well done

b) $M = Ae^{-kt}$

$k = \frac{\ln 10}{51}$ 1m

$\frac{A}{2} = A\,e^{-t \cdot \frac{\ln 10}{51}}$ 1m

$t = 15.35$ $\frac{1}{2}$m

$= 15$ minutes (nearest min) $\frac{1}{2}$m

A few students wrote

$K = \frac{-\ln \frac{9}{10}}{51}$

and

$t = 336$ min

get 2m

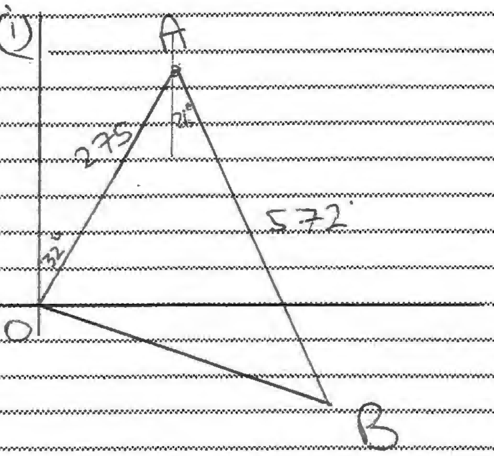
2U MATHEMATICS: Question 8

Suggested Solutions	Marks	Marker's Comments
(i) In $\triangle$s ABM, APD	½	
$A\hat{B}M = A\hat{P}D$ (both $90°$)	½	
$B\hat{M}A = P\hat{A}D$ (alternate angles are equal BM ∥ AD)	½	
$\therefore \triangle ABM \,\vert\vert\vert\, \triangle APD$ (equiangular)	½	
(ii) $\frac{PD}{AB} = \frac{AD}{AM}$ (corresponding sides in similar triangles are in the same ratio)	½	$-\frac{1}{2}$ if ratio is written incorrectly.
$\frac{PD}{40} = \frac{60}{50}$	½	
$PD = \frac{6}{5} \times 40$ ½: $AB^2 + BM^2 = AM^2$, $30^2 + 40^2 = AM^2$, $AM = 50$ (by Pythagoras)		
$PD = 48$ cm	½	
(iii) $AP^2 + PD^2 = AD^2$ (by Pythagoras)	½	(or could have done it using ratio of corresponding sides in similar triangles)
$AP^2 + 48^2 = 60^2$	½	
$AP^2 = 1296$	½	
$AP = 36$ cm	½	
(iv) Area PMCD $= A_{ABCD} - A_{ABM} - A_{APD}$	1	(½ off for each error).
$= (60 \times 40) - \left(\frac{40 \times 30}{2}\right) - \left(\frac{48 \times 36}{2}\right)$	1	
$= 936$ units^2	1	
or PM = AM − PA $= 50 - 36$ $= 14$	1	
$\text{Area}_{DCM} = \frac{1}{2} \times 30 \times 40 = 600$.	½	
$\text{Area}_{PMD} = \frac{1}{2} \times 14 \times 48 = 336$	1	
$\therefore \text{Area}_{PMCD} = \text{Area}_{DCM} + \text{Area}_{PMD}$ $= 600 + 336$ $= 936$ units^2	½	

2U **MATHEMATICS: Question** 8

Suggested Solutions	Marks	Marker's Comments

(b)(i)



$\frac{1}{2}$ — If all of the information is correct!

(ii) $OB^2 = 275^2 + 572^2 - 2(275)(572)\cos 58^\circ$ — $\frac{1}{2}$

$= 402809 - 314600\cos 58^\circ$

$= 402809 - 166712.6005$

$= 236096.3995$

$\therefore OB = 485.8975195$ — $\frac{1}{2}$

$OB = 486\text{ km}$ (nearest km)

$572^2 = 275^2 + 485.89^2 - 2(275)(485.89)\cos A\hat{O}B$

$\therefore \cos A\hat{O}B = \dfrac{275^2 + 485.89^2 - 572^2}{2(275)(485.89)}$ — 1

$A\hat{O}B = 93^\circ 19'$

* If they used the sine rule and didn't find the obtuse angle, they lose $\frac{1}{2}$ mk + lose $\frac{1}{2}$ mk from bearings → 1 off the total.

$\therefore$ Bearing is $(93^\circ + 32^\circ)T$

$\doteqdot 125^\circ T$ (nearest degree) — $\frac{1}{2}$

Question 9.

a (i)

$$\ddot{x} = \frac{dv}{dt}$$

$$= \frac{d}{dt} 3\left[1+9t\right]^{1/2}$$

$$= \frac{3}{2} \cdot 9 \left(1+9t\right)^{-1/2}$$

$$\ddot{x} = \frac{27}{2\sqrt{1+9t}}$$ ✓

[1 mark]

Do not use decimals

$$\frac{13.5}{\sqrt{1+9t}}$$

ii)

$$x = \int 3\left(1+9t\right)^{1/2} dt$$

$$x = \frac{2}{9}\left(1+9t\right)^{3/2} + C.$$ ✓

when $t=0$, $x=-1$

$$-1 = \frac{2}{9} + C$$

$$C = -\frac{11}{9}$$

$\therefore$ Displacement $x = \frac{2}{9}\left(1+9t\right)^{3/2} - \frac{11}{9}$ ✓

[2 marks]

b(i) Deposit $= 20\%$ of $\$500000$

$= \$100000$ ✓ [1 mark]

ii) loan for $\$400000$

Monthly interest $= \frac{8}{12}\% = \frac{8}{1200} = \frac{1}{150}$

Had to show $\frac{8}{12}\%$ to gain a mark.

Amount owing after 1st month.

$$\$400\,000 \times \left(1+\frac{8}{1200}\right) - R$$

$$= 400\,000\left(\frac{1+150}{150}\right) - R$$

$$= 400000\left(\frac{151}{150}\right) - R$$ ✓

[1 mark]

iii)

$$A_2 = 400\,000\left(\frac{151}{150} - R\right)\frac{151}{150} - R$$

$$= 400000\left(\frac{151}{150}\right)^2 - R\left(1+\frac{151}{150}\right)$$

$$A_3 = \left[400\,000\left(\frac{151}{150}\right)^2 - R\left[1+\frac{151}{150}\right]\right]\frac{151}{150} - R$$

$$= 400000\left(\frac{151}{150}\right)^3 - R\left[1+\frac{151}{150}+\left(\frac{151}{150}\right)^2\right] - R$$

$\therefore$ Amount after n^{th} month

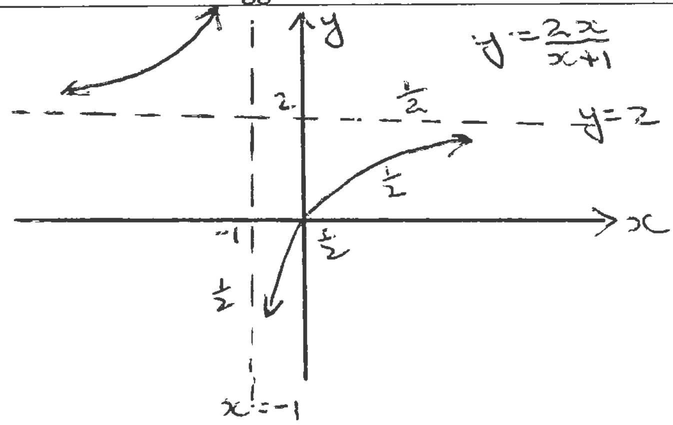
$$A_n = 400000\left(\frac{151}{150}\right)^n - R\left[1+\frac{151}{150}+\left(\frac{151}{150}\right)^2 + \cdots \left(\frac{151}{150}\right)^{n-1}\right]$$ ✓ ✓

To show the answer you need to get a pattern for at least 3 months.

[2 marks]

iv Find monthly repayment $A = 0$ and $n = 20 \times 12 = 240$ $0 = 400000\left(\frac{151}{150}\right)^{240} - R\left[1 + \frac{151}{150} + \left(\frac{151}{150}\right)^2 + \cdots \left(\frac{151}{150}\right)^{239}\right]$ $R = \dfrac{400000\left(\frac{151}{150}\right)^{240}}{\dfrac{\left(\frac{151}{150}\right)^{240} - 1}{\frac{151}{150} - 1}}$ ✓ $= \dfrac{400000\left(\frac{151}{150}\right)^{240} \cdot \frac{1}{150}}{\left(\frac{151}{150}\right)^{240} - 1}$ $= \$3345.76$ ✓	2 marks Round off to 2 decimal place
v) $A_{144} = 400000\left(\frac{151}{150}\right)^{144} - 3345.76\left[\dfrac{\left(\frac{151}{150}\right)^{144} - 1}{\left(\frac{151}{150} - 1\right)}\right]$ $= \$236672.36$ ✓	1 mark
vi) Value of land value $= 270000(1.06)^{12}$ $= \$543293.05$ ✓	1 mark
vii) Man can sell land as $\$543293.05 > \$236672.36$. ✓	1 mark.

2010 TRIAL 2U **MATHEMATICS: Question** 10

Suggested Solutions	Marks	Marker's Comments
10 (a) 	2	Shape $\frac{1}{2}$ mark y intercept labelled gained $\frac{1}{2}$ mark except when there were more than 2 mistakes on the graph.
(b) $\left\lvert \frac{2x}{1+x} \right\rvert < 1$ $\frac{2x}{1+x} = 1$ or $\frac{2x}{1+x} = -1$ $x = 1$ ($\frac{1}{2}$) $\quad 3x = -1$ $x = -\frac{1}{3}$ ($\frac{1}{2}$) $\therefore$ For sum to infinity ($\frac{1}{2}$) $-\frac{1}{3} < x < 1$ [N.B. $x = 0$, $S = 0$]	2	
(c) $S_\infty = \frac{a}{1-r}$ where $a = x$, $r = \frac{2x}{1+x}$ $= \frac{x}{1 - \frac{2x}{1+x}}$ $= \frac{x(1+x)}{1+x-2x}$ $\therefore S_\infty = \frac{x^2+x}{1-x}$	1 1	
(d) $S = \frac{x^2+x}{1-x}$ $\frac{dS}{dx} = \frac{(1-x)(2x+1) - (x^2+x)\times(-1)}{(1-x)^2}$ $= \frac{2x+1-2x^2-x+x^2+x}{(1-x)^2}$ $\therefore \frac{dS}{dx} = \frac{-x^2+2x+1}{(1-x)^2}$	1 1	

2010 TRIAL 2U **MATHEMATICS: Question** 10.

Suggested Solutions	Marks	Marker's Comments
(e) $\frac{dS}{dx} = \frac{-x^2+2x+1}{(1-x)^2}$ For minimum $\frac{dS}{dx}=0$ $\therefore x^2-2x-1=0$ $x = \frac{2\pm\sqrt{4+4}}{2}$ $= 1\pm\sqrt{2}$	1	If the question was made easier by using any other domain except $-\frac{1}{3}<x<1$, max of 2 marks, BUT needed to test endpts of their domain to gain the 2 marks.
But $-\frac{1}{3}<x<1$ from (b) But $1-\sqrt{2}<-\frac{1}{3}$ and $1+\sqrt{2}>1$ then	1	
Since S_∞ is continuous for $-\frac{1}{3}<x<1$ then the minimum occurs at the endpoints. Test end pts. Minimum $= \lim_{x\to -\frac{1}{3}} S = \frac{(-\frac{1}{3})^2-\frac{1}{3}}{1+\frac{1}{3}}$ and $\lim_{x\to 1} S \to \infty$ $= \frac{1-3}{9+3}$ $= -\frac{1}{6}$	2	When $x=1-\sqrt{2}$ an answer of $2\sqrt{2}-3$ ($-0.1715\ldots$) with working received $1\frac{1}{2}$ as question had been greatly simplified i.e. when $x=1-\sqrt{2}$ $S_\infty = \frac{(1-\sqrt{2})^2+1-\sqrt{2}}{1-(1-\sqrt{2})}$ $= \frac{1-2\sqrt{2}+2+1-\sqrt{2}}{\sqrt{2}}$ $= \frac{4-3\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{4\sqrt{2}-6}{2}$ $= 2\sqrt{2}-3$